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THE MEASURE OF THE EARTH:

BEING

An Account of ſeveral OBSERVATIONS made for that Purpoſe by divers MEMBERS of the Royal Academy of SCIENCES at *PARIS*.

Tranſlated out of the *French* by *Richard Waller*, Fellow of the ROYAL SOCIETY.

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THE MEASURE OF THE EARTH.

ARTICLE I.

THE attempt to determine the Magnitude of the Earth is not new. Many ancient Authors have made themselves famous by this enquiry. But the most memorable Attempt for this purpose was that of the *Arabians*, thus Recorded by their Geographer. A great Circle on the Earth is divided into 360 parts, as we also suppose those in the Heavens. *Ptolomy* Author of the *Almagest*, and many other of the Ancients have observed what space upon the Earth contains one of these 360 Parts or Degrees, and have found it to contain $66\frac{2}{3}$ Miles. Those which succeeded them, willing to satisfie themselves by their own experience, met by the order of *Almamon* in the Plains of *Sanjar*, and having taken the height of the Pole, they divided into two Tropps, the one marching as directly as was possible towards the North, and the other towards the South, till the one found the Pole one Degree more, and the other one Degree less elevated; then meeting again at their first station to compare their Observations, they found the one had computed $56\frac{2}{3}$ Miles, but the other just 56. but they agreed to account $56\frac{2}{3}$ for one Degree, so that between the Observations of the Ancients, and of these Moderns there is a difference of 10 Miles.

Abulfeda in his Preface.

Now *Ptolomy* having establish'd the bigness of a Degree 500 Stadia, for which the *Arabs* account $66\frac{2}{3}$ Miles, it follows that the Arabian Mile was equal to $7\frac{1}{2}$ Stadia; but we are to seek what Stadium *Ptolomy* means; for if it were the Greek, eight of which made one ancient Italian Mile, the proportion of the Arabick Mile, so the Italian will be as 15 to 16, and consequently the $56\frac{2}{3}$ Miles found in a Degree by the *Arabs*, will make but $53\frac{1}{8}$ old Italian Miles. But if more favourably to the *Arabs*, we suppose (which

is

is most likely) that the 500 Stadia of *Ptolomy* were the Alexandrian, bigger than the Grecian, according to the proportion commonly received of 144 to 125, we shall find that the Degree measured by the *Arabs* was 61¼ Italian Miles, which makes 47188 Toyses of *Paris*, supposing that the old Roman Foot (the same which Father *Ricciolus* after *Vilalpandus* would have established it) was to that of *Paris* as 667 to 720. though the Roman Foot, of which the Module is to be seen in the Capitol, is to the same Parisian Foot, but as 653 to 720. or thereabouts.

'Tis very remarkable that anciently the measure of the Earth was always upon the diminishing. For if we will believe *Aristotle*, or the most part of the *Mathematicians* of his time, according to his report, a Degree was about 1111 Stadia, whereas *Eratosthenes* counted but 700. *Possidonius* 666, and in fine *Ptolomy* 500. In like manner the *Arabs* following the same example make a Degree less than all that preceded them. But without entering upon the determination, whether these Opinions are so different as they appear, it may suffice in brief to say that we are ignorant of the just quantities of the ancient Measures, all the Measures that the Ancients have left us being altered by time.

Amongst the Moderns, *Fernelius* and *Snellius* are the chief, who not contenting themselves with uncertain Traditions, were willing to leave us their particular Observations for the bigness of a Degree.

Fernelius at the beginning of his *Cosmotheoria* says, that leaving *Paris* he went directly North, until by the Meridian Altitudes of the Sun he found the heighth of the *Pole* one whole Degree more than at *Paris*. But whether because he would imitate the *Arabs*, or for some other Reason he has concealed the name of the place where he staged, saying only that it was at 25 Leagues from *Paris*, and that for knowing this distance more precisely he went in a Coach, and counted all the turns of the Wheel till he arrived at *Paris*. And in fine, having estimated how much the irregularities and turnings of the way might augment the length, he judged that a Degree of a great Circle of the Earth contained 68096 Geometrical Paces, which according to our way of measure are equal to 56746 Toyses and four Feet of *Paris*.

Snellius took a more certain way, and somewhat like what will be found practiced in the following account; for instead of relating his estimation, he searched by Geometrical ways the Meridional Distances between the parallels of *Almain*, *Leyden*, and *Bergopson*, then according to the differences of the heights of the Pole in those Places, he concluded a Degree was 28500 Rhinland Perches, which make 55021 Toyses of *Paris*.

This last Measure was commonly followed as the most exact. But Father *Riccioli* by a method which we shall anon examine, hath (since highly prised above other) made the Degree 64363 Paces of *Bologna*, or about 62900 of our Toyses.

In

In this diversity of Opinions 'twas worth while to try the whole anew for the solution of this famous Problem, not only for the use of Geography in what concerns the difference of Longitudes, but more particularly for the use of Navigation. And that so much the rather, for that to this time not a Person has understood the prevalency of the great advantage that may be made of Telescopes from the executing of this Design, and for that by other means it is easie to establish a measure which cannot change.

ARTICLE II.

THE Earth and Water make but the same one Globe which comprises both the one and the other under the name of the Earth. We shall not stay to shew the proofs here, but this truth being supposed for constant, 'tis demanded what is the bigness of the Globe of the Earth; and since it would be impossible to measure the compass intire, 'tis reduced to the measure of one part, from whence the bigness of the whole may be concluded; which reduction is ordinarily to the quantity of one Degree.

For since the roundness of the Earth is a little varied by the inequality of the Mountains, like that of a very fine Orange by the grain of its Peel; these inequalities are so considerable to our purpose, and so great in comparison of common measures, that for the obtaining of the knowledge of a considerable distance, though less than that of a Degree, 'tis necessary to have recourse to Geometry, to make use of a Chain or succession of Triangles united together, the sides of which are as so many great measures, which passing over the inequalities of the surface of the Earth, give us the measure of a Distance, which it would be impossible to measure otherwise.

For the well forming of these Triangles 'twas necessary to point at far distant Objects with such preciseness, as not only to be sure of directing at the whole Object, but even at a certain point thereof. There has been invented for this divers sorts of sights, but all imperfect and incapable of giving the preciseness requisite. 'Twas on this account *Snellius* willing to excuse the errour of some minutes which he found in his Triangles, had reason to blame his sights, through which (as he says himself) an Object of the bigness of some minutes appeared but as a point, and even so with difficulty. But for some Years it has been thought adviseable to put Telescopes in the place of the old way of Sights, which has been so happily performed that there seems to be nothing more to be desired for this purpose, as will appear by the sequel.

Eratosth. Batavus, pag. 169.

ARTICLE III.

IN the design which was proposed for performing the mensuration of the Earth, it was judged that the space contained between *Sourdon* in *Piccardie*, and *Malvoisine* in the Confines of the *Gastinois*, and of the *Hurepois*, would be very proper for the execution of this design, because these two bounds which are distant one from the other about 32 Leagues, are scituated very near in the same Meridran; and 'twas known by divers Journeys purposely made, that they might be joyned by Triangles, with the high-way from *Villejuive* to *Juvisy*; which way being paved in a strait line, without any considerable inequality,and of such a length (as will appear hereafter) was proper to serve for the fundamental Base of all the Measure that was undertaken.

For actually measuring the length of this way, four Pike Staves, each of two Toyses were made choice of, which being joyned two and two at the great ends by a Screw, made two Measures each of the length of four Toyses.

The manner observed in the measuring was, that after one of the Measures was placed on the Earth, the other was joyned to it end to end, along by a great Rope, then the first was taken up, and so successively. And for the more easy keeping the account, the Measurer who laid the second Rod had ten little stakes given him, one of which he left standing at the head of his Rod every time he laid it on the ground, so that every such stake noted eight Toyses; and when all the ten were taken up, they marked eighty Toyses.

In this manner the distance between the middle of the Mill of *Villejuive* all along the great or high way to the Pavillion of *Juvisy* was twice measured, which distance was found to be 5662 Toyses and four Foot in going, and 5663 and one Foot in returning. But as a nearer approach to exactness could not be hoped, so the difference was divided, and the round number of 5663 Toyses was agreed on for the length of the line, or fundamental Base upon the which we have built all the Calculations hereafter, save only that at the conclusion of our work we verify'd the whole by a second Base of 3902 Toyses actually measured as the former. In which without doubt we had very much the advantage of all those that have preceded us. For *Snellius* having begun by a distance measured of 326 Verges and 4 Foot of the Rhein Measure, which make 630 of our Toyses; It was afterward regulated by one which was not above 87 Rhein Verges, or 168 Toyses. And Father *Ricciolus* framed all his Measure upon a Base of 1088 Bologna Paces, or about 1064 Toyses of *Paris*.

ARTICLE IV.

THE Toyſe of which we ſpeak, and which we have choſen as the moſt certain Meaſure, and moſt uſed in *France*, is that of the Grand Chaſtelet of *Paris*, according to the original which has been lately re-eſtabliſh'd. It is of ſix Foot, the Foot contains twelve Inches, and the Inch twelve Lines; but to prevent, that what has happen'd to all ancient Meaſures (of which nought but the names remain) might not happen to ours; we have adapted it to an Original taken from Nature it ſelf, which ought therefore to be invariable and univerſal. To that effect the length of a ſingle Pendulum was by two great Pendulum Clocks exactly determined, each of whoſe ſingle vibrations or free agitations was one ſecond of time conformable to the mean motion of the Sun, which length was found to be 36 Inches, 8 Lines and a half, according to the *aforeſaid* meaſure of the Chaſtelet of *Paris*.

'Tis commonly known, that to make a ſimple Pendulum, a little ball about the bigneſs of a Muſquet Bullet,is ſuſpended by a very flexible thread, and the length of this Pendulum muſt be meaſured from the top of the thread to the center of the Ball, ſuppoſing the Diameter of the Ball not much to exceed the 36*th* part of the length of the thread, otherwiſe there muſt be an account had of a proportional part which We have here neglected; and care muſt alſo be taken that the vibrations be ſhort, for if they be beyond a certain Degree, they are of unequal duration one to another.

The Ball of our Pendulum was of Copper of an inch in Diameter, and it was turned. The thread with which the firſt experiments were made was of flat or raw ſilk. But becauſe that ſtretches ſenſibly by the leaſt humidity of the air, it was found that 'twas better to uſe a ſingle filament of a ſort of long Flax called *Pite*, which is brought out of *America*. The upper end of the thread was put between a ſmall Vice with a ſquare head, which held it faſt ſcrewed moſt exactly; by this means the motion of the Pendulum was more free, and the length more eaſily meaſured by an Iron Rod exactly fitted between the end of the Vice and the Ball.

The two Clocks made uſe of were of the greater ſort, whoſe Pendulums meaſured whole ſeconds, they were exactly regulated according to the mean motion of the Sun, and went ſlower by 3 Minutes 56 ſeconds at every return of the ſame fixt Star to the Meridian, with ſuch a regularity that ſometimes they differed not one from another by one ſecond during many Days. A ſingle Pendulum was ſet in motion, and made to go and come from the ſame ſide as the Pendulums of the Clock did, and being left in this condition they were inſpected from time to time to ſee how they went. For how little ſoever the length of this ſingle Pendulum either exceeded or wanted of 36 Inches, 8½ Lines, one might perceive ſome diſagreement in leſs than an hour. 'Tis true that this length was

not always found ſo preciſe, and that it ſeemed that it ought to have been regularly a little ſhortned in Winter and lengthened in Summer. But that however was but the 10*th* part of a Line) ſo that having a reſpect to this variation, it has been judged beſt to take the mean between them, and to take the length of 36 Inches 8½ Lines for the certain Meaſure.

If the length of the Pendulum for ſeconds be once found expreſt according to the uſual Meaſure of every place, by this means may be had the proportion of the different Meaſures ſo exact as if the originals had been compared, and this advantage would thence accrue, that for the future any change therein might be diſcovered.

But beſides the particular Meaſures, an agreement might be found of ſuch as follow, which will need no other original but the Heavens.

The length of a Pendulum of a ſecond of the middle time might be called by the name of an Aſtronomical Ray, the third of which ſhall be the univerſal Foot. The double of the Aſtronomical Ray makes the univerſal Toiſe, which will be to that of *Paris* as 881 to 864.

Four times the Aſtronomical Ray may make the univerſal Perch equal to the length of a Pendule of two ſeconds.

Finally the univerſal Mile may contain 1000 Perches.

Theſe univerſal Meaſures ſuppoſe that the difference of places cauſeth no ſenſible variation to the Pendulums. 'Tis true, there have been made ſome experiments at *London*, *Lyons* and *Bolognia* in *Italy*, by which it ſeems one might conclude that the Pendulums ought to be ſhorter in ſome proportion as the Æquinoctial is approacht. Conformable to a conjecture which has been formerly propoſed in the Aſſembly, that ſuppoſing the motion of the Earth, weights ought to deſcend with leſs power under the Æquinoctial than under the Poles. But we are not ſufficiently informed of the juſtneſs of theſe Experiments to make any concluſion thence. And we muſt beſides note, that at the *Hague*, where the heighth of the Pole is greater than at *London*, the length of a Pendulum exactly determined by means of Clocks, was found the ſame as at *Paris*. 'Tis for this we adviſe thoſe who would make experiment with a ſingle Pendulum, to make uſe of great Pendulum Clocks, for that otherwiſe they will difficultly meet with the juſt Meaſure. If it ſhould be found by experience that the Pendulum will be of different lengths in different places, the ſuppoſition we have made concerning the univerſal Meaſure drawn from the Pendulums, cannot hold, but this hinders not but that in every place there will be a perpetual and invariable Meaſure.

The length of a Pariſian Toyſe, and that of a Pendulum of ſeconds, ſuch as we have now eſtabliſht, will be carefully preſerved in the Magnificent Obſervatory, which His Majeſty has cauſed to be built for the advancement of Aſtronomy.

AR-

page. 7

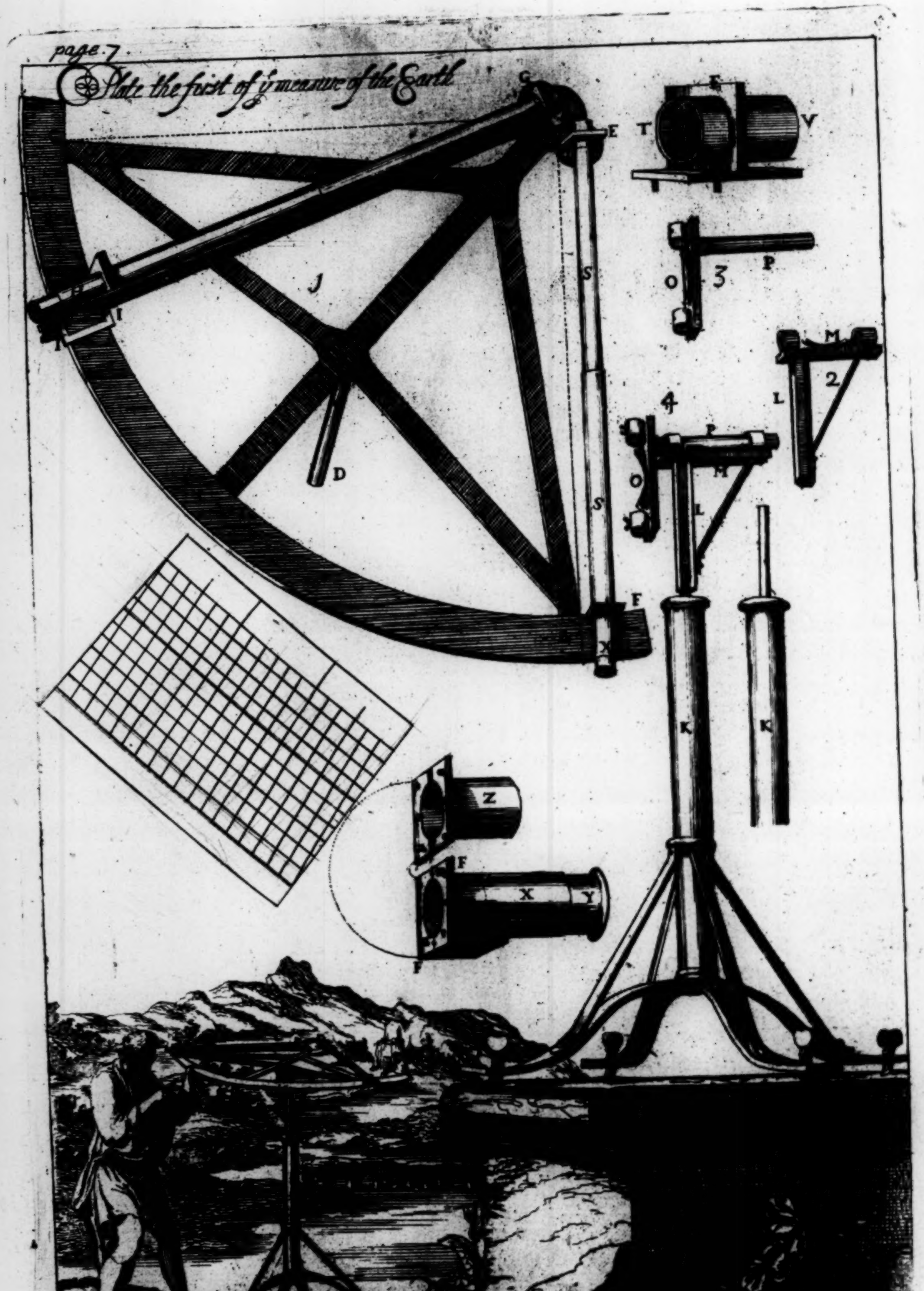


Plate the first of ye measure of the Earth

ARTICLE V.

SINCE the Instrument we made use of for measuring the Earth, had somewhat singular, it will not be insignificant to describe it before we come to the following Observations.

This Instrument was a quarter of a Circle of 38 Inches Radius, the body of it is of Iron, and all the pieces are fastned together underneath by Screws upon the Area of it. The Limb B C and that part about the Center A, are covered with Copper. The Broach or Cilinder D is fastned perpendicular to the back of the Instrument to fix it on its Pedestal. E F is a Telescope which serves instead of the immovable sights, being fastned at one end to the Plate of the Center A, and at the other end to one of the extremities of the Limb. Plate the first.

G H is another Telescope carried by an Alidade or arm of Iron which turns upon the Center A, and which may be fixed upon any part of the Limb desired, according to the Angle to be observed.

The Limb B C is exactly divided even into Minutes very distinctly, much of the bigness and form represented in the adjoining Figure.

An Hair stretched in the little frame I, or a silver Wire smaller than a Hair, serves for the fiducial Line of the Alidade, by which one may very easily distinguish to the fourth part of a Minute, especially if a Loupe or Glass that magnifies the object, be used.

But that which we have here principally to describe, is the construction of the Telescopes E F and G H, which being in all things alike the one to the other, it will be sufficient to describe one of them.

S S is a Cylinder of Latton or Tin, made of two pieces running one within the other, that they may be taken off or put on at pleasure upon the two Pinnules E F which are fixed.

The Object Pinnule E carries in the fore-part of it marked T, an Object Glass of a Telescope of a length proportioned to the Instrument: And by the side V it sustains one of the ends of the Cylinder S S.

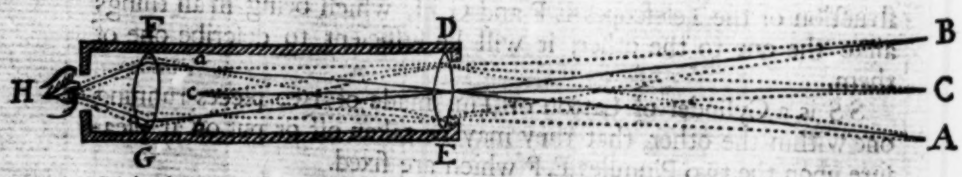
The eye Pinnule F is of three Pieces, the first F X which is fastned to the Limb of the Instrument is a hollow Cylinder about 3 Inches long, sodered to the middle of the (Chasse) or Frame F F, upon the face of which are two small single Clews of black Silk stiff strained at right Angles in four small graved strokes, which keeps them from breaking, and they are fastned by the means of a little melted Wax. The second Z is a little hollow Cylinder sodered as the former to the middle of a square Piece, which by two Screws is joined to the frame F F, to serve as well for the defence of the Filets, as to sustain the great hollow Cylinder S S. The third Y is another

another little hollow Cylinder which is ſlipped within the firſt X; and which carries the Eye-Glaſs of the Teleſcope.

The fixed diſtance between the two Pinnules E. F ought to be ſuch that the anterior face of the frame F F, where the Filets of the Teleſcope are ſtrained, do meet each other exactly in the focus of the Object Glaſs; and this neceſſity cauſes the Object Glaſſes to be made (for the moſt part) firſt before the Inſtrument is begun. All put together does the effect of a Teleſcope that inverts the Object, which inconvenience may eaſily alſo be rectified, making uſe of more Eye-Glaſſes, and a little uſe will make it inſenſible.

All the peices of ſuch a Teleſcope as it here deſcribed, are yet more fully repreſented in the fourth Plate.

Beſide the advantage that the common Teleſcopes give of being able better to diſtinguiſh a diſtant Object, they do alſo much facilitate the ſetting it true pointing to the Object with all the preciſeneſs imaginable; for after one has through this Teleſcope taken notice of the far diſtant Object, one may at the ſame time ſee very diſtinctly the Threads (or Wires) that are in the Teleſcope, and alſo all that which the ſaid Threads hinder to be ſeen of the Object, as if they were indeed ſtretched upon the very object it ſelf, and the Eye upon removing perceives no Paralax at all between the one or the other, provided the Fillets or Threads, as we have ſaid, be placed exactly in the focus of the Object Glaſs, becauſe 'tis in this place that the painting of the Object is made reverſed, which comes immediately to our Eye, and which is the place of the immediate Object, as may be eaſily underſtood by the following Figure.



A B C are three points of an Object, every of which fill the Object Glaſs D E of the Teleſcope F D E G with Rays; all theſe Rays having paſſed, traverſe the Glaſs D E, they proceed to reunite by order in three other points a b c, namely, thoſe of A in a, of B in b, and of C in c; then theſe ſame Rays are ſeparated again, and proceed to fall upon the Eye-Glaſs, which in fine turns them towards the Eye H, the Rays of C are not continued to the Eye, to the end that it may appear what muſt happen when it meets with an obſtacle in ſome part of the focus as in c, becauſe it is evident that this obſtacle hinders all the Rays of the point C, without permitting any one of them to arrive at the Eye, as if one had indeed covered the Object it ſelf at the point C; but this Obſtacle, ſuch as it may be, a ſingle filament of Silk, makes its diſtinct Image in the Eye preciſely

precisely in the place where the Object which it hinders would have made its own Image, because the Eye is altogether disposed for receiving the Rays which are come from the focus a b c travers the Eye Glass F G.

It is to be added hereto, that since all the Rays of the same point of the Object are reunited in another point of the focus of the Object Glass, it happens here that notwithstanding all the aperture of the Object Glass D E, one has the same exactness for pointing as if the Object Pinnul or sight were but one single, small, and almost indivisible hole through which the point C could traject but one Ray, which might be intercepted by the least obstacle placed in the Line C c, because that which necessitates the placing the Threads in the focus is for that if they are placed either nearer to or farther from the Object Glass, they cannot hinder all the Rays from the same point, which are not elsewhere united but only in the focus, and there will be some Parallax sensible if they be placed out of it, upon changing the position of the Eye, which however is most to be regarded when the aperture of the Object Glass is large, for if it be but small, the place of the Threads does not require so very precise a distance from the Object Glass, because at some distance on either side the focus, either nearer to or further from the Object Glass, the Rays are not so far separated as to become sensible. And 'tis also in the straitning or lessening of the aperture of the Object Glass that an inconvenience may be prevented, which happens to the Threads when being well placed for a remote Object, they are not so exact for Objects that are nearer.

There may remain one difficulty upon the account of the Object Glass, if it be not of an equal thickness, thereby causing some refraction, and bending the principal Ray C c from a straight Line. But notwithstanding all the defects of this Glass, there is no reason to fear in respect of the Angles of position, or of the apparent distances which one would observe, because when the two Telescopes are directed to the same Object at a distance, the fiducial Line of the movable rule (or arm) falls exactly upon the beginning of the first Degree. And this is a proof with which we ought always to begin when one would take Angles. We shall give in the ninth Article the means of remedying defects and refractions of Glasses in regard of heights.

The Figures 2, 3, 4, represent the pieces which serve to set the Quadrant upon its Foot. The piece L M movable upon the Foot K, suffices to set this Instrument to its plumb or perpendicular, when one would observe heights, but for putting it horizontal, the second Piece O P must be added to L M, in the manner as is represented in the fourth Figure, and then one may give the Quadrant such position as one will, as with a Knee.

Thus you have the full description of the Instrument which gave the Angles of position with so much exactness, that upon the whole compass of the Horrison taken at 5 or 6 Angles, there was not

C found

found above a minute more or less than it ought to be, and which often also happened within about 5 seconds of the just account, so that it was not necessary to carry a bigger Instrument, of which it was otherways impossible to make use in several occurrences.

ARTICLE VI.

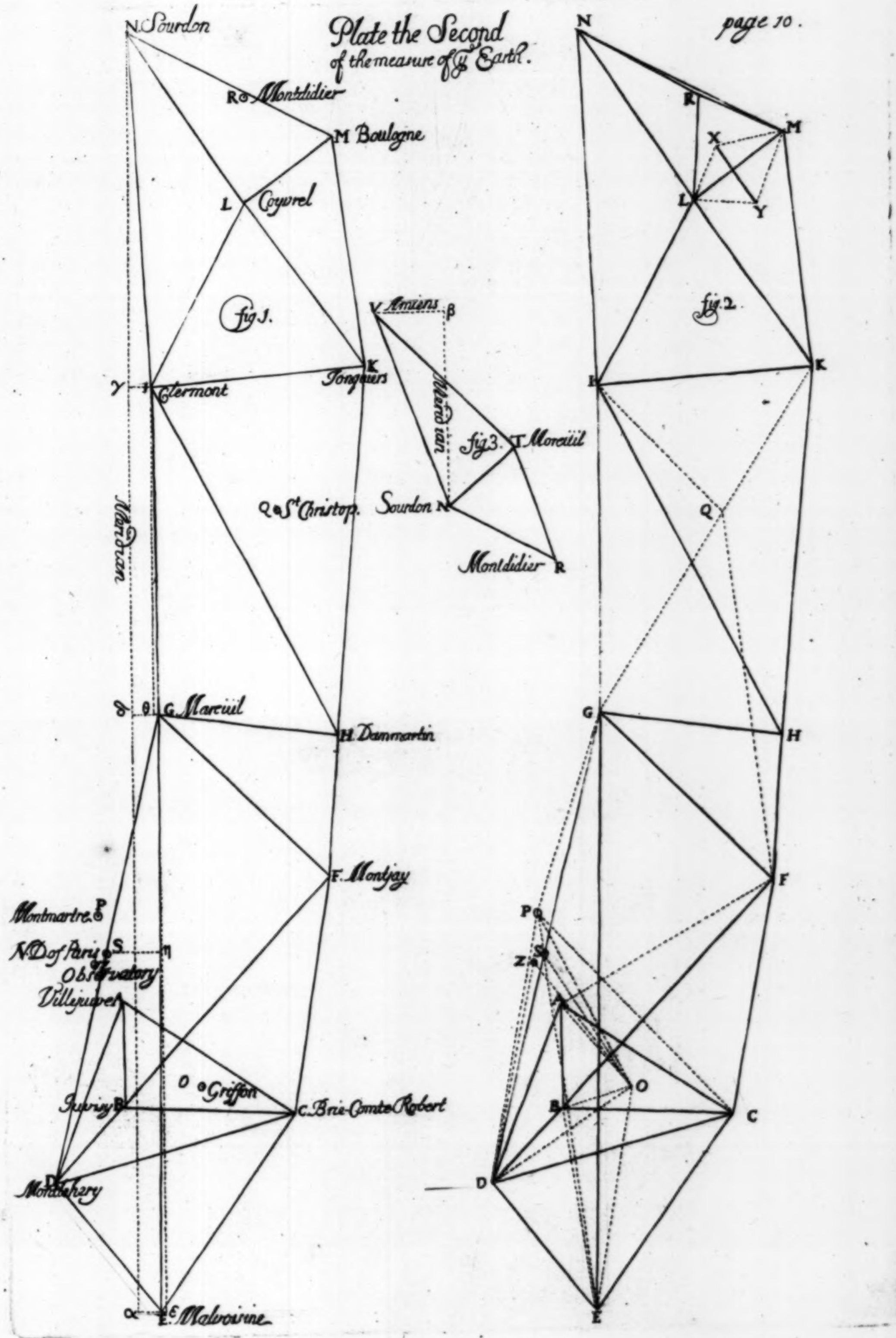
THE distance which was proposed to be measured from *Malvoisine* to *Sourdon*, is found as 'twere parted into three Lines, to wit, from *Malvoisine* to *Mareüil*, from *Mareüil* to *Clermont*, and from *Clermont* to *Sourdon*. These particular distances were known by the means of 13 Triangles, represented in the first Figure of the second Plate. There were two of them which needed no particular Observation, so that one may account but 11 principal Triangles, the other which are represented in the second Figure of the same Plate, having chiefly served for the verification. Here follows the list of Stations and precise Places to which Observations have been made for forming the Triangles.

A *Is the middle of the Mill of* Villejuive.
B *The nearest Coin of the Pavillion of* Juvisy.
C *The point of the Steeple of* Brie-Comte-Robert.
D *The middle of the Tower of* Montlehery.
E *The top of the Pavilion of* Malvoisine.
F *A piece of Wood set up purposely on the top of the Ruines of the Tower of* Monjay, *and made larger with Straw tyed about it.*
G *The middle of the Hillock of* Mareuil, *where 'twas necessary to make a Fire for a mark.*
H *The middle of the great Pavillion in the Oval of the Castle of* Dammartin.
I *The Steeple of* S. Samson *of* Clermont.
K *The Mill of* Jonquiers *near* Compiegne.
L *The Steeple of* Coyvrel.
M *A little Tree upon the Mountain of* Boulogne *near* Montdidier.
N *The Steeple of* Sourdon.
O *A little forked Tree upon the But of* Griffon, *near* Villeneuve S. Georges.
P *The Steeple of* Montmartre.
Q *The Steeple of St.* Christopher's, *near* Senlis.
AB *Is the first Base actually measured, of 5663 Parisian Toyses.*
XY *Is a second Base of 3902 Toyses, actually measured as the former.*

It can't be imagined that 'twas possible to place a large Quadrant at the point of Steeples, and of such other Places as we made choice of for forming exactly the Triangles.

But that we might have a remedy for this, we always had a care to observe the apparent thickness of Objects towards which we

Plate the Second
of the measure of y^{e} Earth.



we directed. For example, in pointing at a Tower we were not content to have taken only the middle, but of how many minutes and seconds its thickness appeared, which gave means afterwards to set the Instrument on what part one would of the same Tower, in case the middle were imbarassed or inaccessible.

'Tis true that with all the precaution that could be taken, and after turning the Instrument two or three times towards the same station, 'twas impossible sometimes to avoid the error of some seconds upon the summ of the three Angles of the same Triangle; in which case there can be no difficulty of correcting the Triangle without fear of any considerable error, because all the Angles were large, and there was always some one of which there was not so much certainty as of the rest, and upon which the fault ought to be laid. The principal Corrections that were made are remarked.

In the List of the Triangles this Rule is kept, of not giving any Angle that was not observed by the Quadrant before described, and to omit those which we were obliged to conclude, though in effect there was no great difference to be made between the one and the other, because of the great preciseness with which they were directed at, and of the great care that was taken not to err in the quantity of the Angles that were observed, by reiterating several times the Observations of the same Angle, and the causing them to be made by several Observers who kept their Memoirs apart. Besides that, in the first courses that were made for the discovery of proper stations, all the Angles generally had been observed; and tho these were with lesser Instruments, which gave the minutes but by six and six; yet they were not hindered from coming to so much exactness as was necessary to make it appear that they did not all fail or err in the Conclusions.

The First Triangle A B C.

To find the side A C.
CAB 54° 4′ 35″.
ABC 95 6 55.
ACB 30 48 30.
AB 5663 Toyses actually (measured
Then AC 11012 Toyses five (Foot
And BC 8954 Toyses.

The Second Triangle, ADC for D C and A D.

DAC 77° 25′ 50″.
ADC 55 0 10.
ACD 47 34 0.
AC 11012 Toyses 5 Foot.
Then D C 13121 Toyses (three Foot.
And A D 9922 Toyses two (Foot.

The third Triangle DEC.

For DE and CE.
DEC 74° 9′ 30″.
DCE 40 34 0.
CDE 65 16 30.
DC 13121 Toyses 3 Foot.
Then DE 8870 Toyses 3 Foot.
And CE 12389 Toyses 3 Foot.

The fourth Triangle.

For DF.
DGF 113° 47′ 40″.
DFC 33 40 0
FDG 32 32 20.
DC 13121 Toyses three Foot.
Thence DF 21658 Toyses.

Note, *That in the* fourth Triangle, *the* Angle DFC *was augmented* 10″ *which were wanting to make up the summ of the three* Angles.

The V. Triangle DFG for DG and FG.
DFG 92° 5′ 20″.
DGF 57 34 0.
GDF 30 20 40.
DF 21683 Toyses.
Thence DG 25643 Toyses.
And FG 12963 Toyses 3 Foot.

From these five Triangles 'twas easie to conclude the distance GE between *Malvoisine* and *Mareüil*, without supposing any new Observation.

The VI. Triangle GDE for GE.
GDE 128° 9′ 30″.
DG. 25643 Toyses.
DE 8870 Toyses three Foot
Thence GE 31897 Toyses.

By the Calculation of the same Triangles were found the Angles DGE of 12° 38′, and DEG of 39° 12′ 30″, the same which they were found also by Observation, which may serve as a proof for GE. And it ought to be considered, that as this Triangle is but as a consequence of the preceding, that it has two sides known, and all the Angles well establisht, the smalness of the Angle DGE, cant hinder the certainty of the Conclusion for GE, besides that hereafter the same distance GE shall be verifyed by other Triangles.

Upon the occasion of these Angles DGE and DEG, it was that Fires were made at *Mareüil*, *Montlehere*, and *Malvoisine*. A large Fire of three Foot made at *Mareüil* and seen from *Malvoisine*, appear'd to the Eye like a Star of the third Magnitude.

'Tis

'Tis not our design to draw hence any conjectures concerning the fixed Stars, but only to make the following remark, That if one considers the distance of 31897 Toyses, the Fire which had three Foot of breadth ought to have been seen under an Angle of 3′ 14″, and yet when it was seen with the Telescopes of the Quadrant, of which the Object Glasses were excellent, it was not above half hid or covered by one of the silk Clews which were placed in the focus of the Telescope; now the bigness of this Filament (which was presently measured with a Microscope) was the three hundredth part of an Inch. It follows then that in a Telescope of 36 Inches it takes up the space of about 4″, so that the Fire which it covered but half, took up the space of eight seconds, though it ought in effect have appear'd but of three seconds.

From this Experiment it may be concluded that even with Telescopes, Luminous Objects do appear bigger than they ought. It were well to make trial of this with long Telescopes, which will be reserved for another time.

We have said above that the distance EN was divided into three Lines, the first, namely GE, has been Calculated, but before we pass to the second, 'twill be much to the purpose to verifie all that we have hitherto established by several other Triangles.

Another way for AD by the Triangle AOB.

AOB	62°	22′	0″.
ABO	75	8	20.
BAO	49	29	40.

AB 5663 Toyses.
Thence AO 6178 Toyses (2 Foot.

But by the Triangle AOD.

AOD	76°	50′	0″.
ADO	37	19	20.
DAO	65	50	40.

AO 6178 Toyses.
Thence AD 9922 Toyses (2 Foot.
And DO 9298 Toyses.

Otherwise for DE by the Triangle DOE.

DOE	47°	0′	00″.
DEO	50	2	50.
EDO	82	57	10.

DO 9298 Toyses.
Thence DE 8870 Toyses (5 Foot.
Instead of 8870 Toyses 3 Foot.

Otherwise for CE by the Triangle ACE.

ACE	88°	8′	0″.
AEC	42	22	30.
EAC	47	24	30.

AC 11012 Toyses five Foot.
Thence CE 12388 Toyses two (Foot.
Instead of 12389 Toyses three (Foot.

Yet

Yet otherwise for C E in Triangle B C E.

B C E 57° 19′ 30″.
B E C 44 55 45.
E B C 77 44 45.
B C 8954 Toyses.
Thence E C 12390 Toyses.
The Angle E B C being diminisht 10″.

Otherwise yet for C E in Triangle P D C.

P D C 65° 31′ 0″
P C D 62 2 40.
D C 13121 Toyses three Foot.
Thence P C 15064 Toyses three Foot.
And D P 14621 Toyses three Foot.

But in the Triangle P C E.

P C E 102° 36′ 40″.
P E C 43 9 30.
P C 15064 Toyses three Foot.
Thence C E 12389 Toyses instead of 12389 Toyses three Foot.

Otherwise for D F in Triangle A C E.

A C F 66° 13′ 40″.
A F C 50 33 20.
F A C 63 13 00.
A C 11012 Toyses five Foot.
Thence A F 13051 Toyses.

But in the Triangle F A D.

F A D 140° 38′ 50″.
A F 13051 Toyses.
A D 9922 Toyses.
Thence D F 21657 Toyses three Foot.
For 21658 Toyses.

Otherwise

Otherwise for F G in Triangle GAF.

GAF 52° 8′ 50″.
GFA 75 12 10.
FGA 52 39 00.
A F 13051 Toyses.
Thence F G 12963 Toyses for 12963 Toyses 3 Foot.

The summ of the two Angles A F C, G F A exceed by 10″, that of the two C F D, D F G, which is neglected, because an error so little considerable deserves not the exposing one self a second time to danger in mounting to the top of the Tower of *Monjay* which is half ruined.

Otherwise for G E in Triangle G D C.

G D C 62° 53′ 0″.
D G 25643 Toyses.
D C 13121 Toyses three (Foot.

Thence G C D 86° 24′ 25″.
And G C 22869 Toyses three (Foot.

But in the Triangle G C E having put together G C D and D C E.

G C E 126° 58′ 25″
G C 22869 Toyses three (Foot.
C E 12389 Toyses three (Foot.

Thence G E 31893 Toyses (three Foot.

Instead of 31897 Toyses, but parting the difference we make G E 31895 Toyses.

The VII. Triangle F G H.

For G H.
F G H 39° 51′ 0″.
F H G 91 46 30.
H F G 48 22 30.
F G 12963 Toyses three (Foot.

Thence G H 9695 Toyses.
In this Triangle the Angle G F H is diminisht 10″.

The

The VIII. Triangle G H I.

For G I and I H.
G H I 55° 58′ 00″.
G I H 27 14 00.
I G H 96 48 00.
G H 9695 Toyſes.
Thence G I 17557 Toyſes.
And H I 21037 Toyſes.

Another way for G I in Triangle Q F G.

Q F G 36° 50′ 0″.
Q G F 104 48 30.
G F 12963 Toyſes three Foot.
Thence Q G 12523 Toyſes.
But in the Triangle Q G I.
Q G I 31° 50′ 30″.
Q I G 43 39 30.
Q G I 12523 Toyſes.
Thence G I 17562 Toyſes.
And Q I 9570 Toyſes.

By the Triangle Q H I, G I is found of 17557 Toyſes only, but for a reaſon we ſhall after ſhew, the laſt calculation is followed, which makes G I of 17562 Toyſes, and by conſequence H I 21043 Toyſes.

The IX Triangle H I K for I K.

H I K 65° 46′ 00″.
H K I 80 59 40.
K H I 33 14 20.
H I 21043 Toyſes.
Thence I K 11678 Toyſes.

The ſumm of theſe three Angles being too great by 20″, by which the Angle H K I is diminiſhed, upon which it ſhould be noted that the point H taken for the middle of the great Pavillion on the oval of the Caſtle of *Dammartin* was difficult to determine when obſerved from the ſtation K; and that it may happen in a diſtance of 19436 Toyſes, the Eaſt ſide of this Pavillion appear'd greatned by ſome other adjoyning Objects, which cauſed the Angle H K I to be obſerved bigger than it ought.

Otherwiſe for I K in the Triangle Q I K.
Q I K 49° 20′ 30″.
Q K I 53 6 40.
Q I 9570 Toyſes.
Thence I K 11683 Toyſes.

After that which has been ſpoken concerning the point H, there is cauſe to reſt ſatisfied rather in this laſt Calculation than in that of the Triangle H I K, ſo much the more for that we being aſſured to have pointed moſt exactly at the Steeple of St. *Chriſtopher*, which was ſeen on all ſides like a very fine Needle.

We were not able to place the Quadrant in the Steeple, nor in that of *Coyvrel* for obſerving the Angles, which we were therefore obliged to conclude. But we took ſo much care in obſerving all the other Angles, and the Inſtrument gave the Circuit of the Horiſon ſo exactly, that there ought to remain no doubt at all upon that.

The

The X. Triangle I K L for K L and I L.

L I K 58° 31′ 30″.
I K L 58 31 00.
I L 12683 Toyses.
Thence K L 11188 Toyses two Foot.
And I L 11186 Toyses four Foot.

The XI Triangle K L M for L M.

L K M 28° 52′ 30″.
K M L 63 31 00.
K L 11188 Toyses two Foot.
Thence L M 6036 Toyses two Foot.

The XII Triangle L M N for L N.

L M N 60° 38′ 00″.
M N L 29 28 20.
L M 6036 Toyses two Foot.
Thence L N 10691 Toyses.

The XIII Triangle I L N for N I.

The summ of the Angles I L K K L M M L N, being taken from 360, there remains
I L N 119° 32′ 40″.
But L N 10691 Toyses.
And I L 11186 Toyses four Foot.
Thence I N 18905 Toyses.

So it is that upon the foundation of the first Base A B, which was actually measured, we have concluded the length of the three Lines E G, G I, I N, from *Malvoisine* to *Sourdon*.

But because the four last Triangles were not accompanied with a verification, and because we had a great desire to have a new clearing of the matter upon the VIII and IX Triangles, we judged it necessary to come to an actual measure of a new Base.

The Line of distance L M between *Coyvrel* and the Mountain of *Boulogne* was found the most proper to serve for this last verification, not at all for that this Line could be actually measured, but because it passed a cross a great plain where we had the convenience to take the transversal Base X Y from the Mill of *Mery*, even almost to the Valley of St. *Martin* within a pace of *Mont-dedier*.

Which Base actually measured with the same Pike Staves made use of for the first measuring, and which had been verified all *de novo*, was found of 3902 Toyses. See here the Calculation which was made thereupon.

Of the Triangle X Y L.	But in the Triangle X Y M.
X Y L 50° 37′ 40″.	X Y M 56° 46′ 15″.
Y X L 54 10 45.	Y X M 65 20 45.
X Y 3902 Toyses of actual (measure.	X Y 3902 Toyses.
Thence Y L 3273 Toyses two (Foot.	Thence M Y 4187 Toyses.

In fine, in the Triangle M Y L

M Y L 107° 23′ 55″.
Y L 3272 Toyses three Foot.
Y M 4187 Toyses.
Thence M L 6037 Toyses in-stead of 6036 Toyses 2 Foot.
Then by proportion I N 18907 (Toyses.
And G L 17564 Toyses.
But the E G ought to be left be-cause it has been several ways verified.

That small difference there was found between the distance which was concluded from the first Base, and that which we found by the last, made us see we had reason to suspect the Triangles which butted at the point H, and that those of the point Q had better deserved to pass for the principal. But we had no mind at all to change the order we have kept.

ARTICLE VII.

Though our first design were to terminate all our measures at *Sourdon*, yet we found a necessity as 'twere of continuing them to *Amiens*, where we resolved to go to take the heighth of the Pole for verifying the Calculation of *Fernelius*. We would willingly have had time enough to have fought out in the Plains of *Santerre* some point proper for finishing this measure by two great Triangles. But the Season being already too far advanced, we were fain to content our selves with what we met with in the borderings of *Sourdon*, where it was necessary to stay for taking the heighth of the Pole.

R is

R is the Steeple of St. *Peter* of *Montdidier*. Second Plate 3d Fig.
T a Tree upon the Mountain of *Mareuil*.
V the Steeple of *Nostre Dame d' Amicus*.

In the Triangle L M R.

L M R	58°	21′	50″.
M R L	68	52	30.

L M 6037 Toyses.
Thence L R 5510 Toyses (three Foot.

In the Triangle N R L.

N R L	115°	01′	30″.
R N L	27	56	30.

L R 5510 Toyses three Foot.
Thence N R 7122 Toyses (two Foot.

In the Triangle N R T.

N T R	72°	25′	40″.
T N R	67	21	40.

N R 7122 Toyses two Foot.
Thence N T 4822 Toyses four (Foot.

In fine in the Triangle N T V.

N T V	83°	58′	40″.
T N V	70	34	30.

N T 4822 Toyses four Foot.
Thence N V 11161 Toyses (four Foot.

Some have thought that we ought to have added to all these Calculations the true position of the Towers of *Nostre Dame* of *Paris*, and of the Observatory.

S is a Lanthorn over the stairs of the South Tower of *Nostre Dame* of *Paris*. Second Plate, first and 2d Figures.

Z is the middle of the South Face or Front of the building of the Observatory.

In the Triangle D O S.

D O S	88°	16′	40″.
D S O	46	35	00.
S D O	45	8	20.

D O 9298 Toyses.
Thence D S 12795 Toyses.
And O S 9373 Toyses.

In the Triangle D O Z.

D O Z	82°	5′	10″.
D Z O	51	34	00.
Z D O	46	20	50.

D O 9298 Toyses.
Thence D Z 11757 Toyses.
And O Z 8588 Toyses three (Foot.

ARTICLE VIII.

After having measured the particular Distances between *Malvoisine*, *Mareüil* and *Sourdon*, and having added to those that of *Amiens*, the position of each of these Lines in respect of the Meridian ought to be examined.

First Plate For this purpose in the Month of *September*, 1669, we went upon the Hillock of *Mareüil*, at the place marked G, where we could see *Malvoisine* on the one side and *Clermont* on the other, and placing the Quadrant furnisht with two Telescope sights perpendicular upon its foot, so that the Telescope EF remained always in the level, whilst the plain of the Instrument was turned vertically, and that the Telescope sight of the Alidade GH pointed at the Polar Star. This Star was so followed to its greatest digression, where it remained a very sensible space of time without parting from the vertical filament of the Telescope with which it was observed, then leaving the Instrument fixed in its position the remainder of the night, even until the day was come, we could discover the place on the border of the Horison, to which the Telescope EF was found to point; and determine by this means the vertical of the greatest digression of the Polar Star. For 'twas known by experience, that when the Quadrant was set to its plumb, the two Telescopes always remained pointed in the same vertical.

By this Observation which was divers times reiterated, we were assured of a distant point which markt the vertical Circle of the greatest Oriental Digression of the Polar Star, which vertical made with the line GI an Angle of 4° 55′ towards the East. The complement of the declination of the Polar Star being then 2° 28′, and the height of the Pole on the Hillock of *Mareüil*, as it was afterward found 49° 5′, and by consequence the digression of the Polar Star was 3° 46′, then there remained yet one Degree and nine Minutes, by which the line GI declined from the North towards the West; and because that otherways the lines GI GE make an Angle of 178° 25′ toward the West, which Angle augmented by the declination of the line GI makes but 179° 34′. it followed that GE declined 26′ from the South towards the West.

The following Year in the Month of *October*, there was chosen by *Sourdon* in the line NV, a place in the open Field, whence the Steeple of *Nostre Dame* of *Amiens* could be discovered, and in the manner explained, 'twas observed several times that this line NV declined 18° 55′ from the North towards the West, whence it was easie to conclude that NI declined by 2° 9′ 10″ from the South towards the East.

These last Observations were made in a time wherein the Pole Star was found in its greatest digression a little after Sun set, and thereby we had the convenience of finishing the Observation all at once, without being obliged to leave the Instrument in its position, because

cause 'tis one of the advantages of the Tellescope Sights, that by means of them one may discover the fixed Stars of the second magnitude in the greatest clearness of the *Crepusculum*, and that those of the first Magnitude may be observed in full Sun-shine, which will be a great help to Astronomy; we have made several curious Observations, which we shall hereafter Publish.

If we suppose then that the Meridian Line of *Sourdon* be prolonged toward the North, till it meets the parallel of *Amiens* at the point β for the making the Rectangle Triangle N β V, the Angle of Declination V N β, being 18° 55′ and the hypothenuse N V, being found 11161 Toyses, 4 Foot, it follows that the Meridian Distance N β between the parallels of *Sourdon* and *Amiens* is 10559 Toyses, 3 Feet, and that the Arch of the Parallel V β comprised between *Amiens* and the Meridian of *Sourdon* is 3617 Toyses, 4 Foot. Second plate, third Figure.

After the same manner if we suppose that the same Meridian Line of *Sourdon* be prolonged towards the South, till it meets with the Parallel of *Malvoisine* at the point α, and that this Meridian be divided into three parts by the perpendiculars G δ I γ which represent the Parallels of *Mareüil* and *Sourdon*, that moreover the particular Meridian Lines of those places be drawn, to wit, G ε, from *Mareüil* to *Malvoisine*, and I θ from *Clermont* to *Mareüil*. Second Plate, first Figure.

In the Triangle N γ I, rectangled in γ.	In the Triangle G I θ, rectangled in θ.
N I 18907 Toyses.	I G 17564 Toyses.
γ N I 2° 9′ 10″.	G I θ 1° 09′ 00″.
Thence N γ 18893 Toyses, 3 Foot.	Thence I θ or γ δ, 17560 Toyses, 3 Foot.
And γ I 710 Toyses.	And G θ 352 Toyses.

In the Triangle G E ε rectangled in ε.

G E 31895 Toyses.
E G ε 00° 26′ 00″.
Thence G E or δ α, 31894 Toyses.
And E ε 241 Toyses, 3 Foot.

The 3 lines N γ, I θ, G ε, make together the whole Distance between the Parallels of *Sourdon* and of *Malvoisine*, of 68347 Toyses, 3 Foot;

3 Foot; to the which Distance adding that between the Parallels of *Sourdon*, and of *Amiens*, which has been found of 10559 Toyses, 3 Foot, we have the Distance between *Malvoisine*, and the Parallel of *Amiens* of 78907 Toyses: And tho in effect the four Lines of which this whole Distance is composed, are as it were the sides of a Polygon, which one would describe about the Earth; and that 'tis true in Geometrical Rigor, that the compass of such a Polygon is bigger than the circumference of the Earth; yet is it notwithstanding so little different in this case, that 'twill be to no purpose to take notice of it; since the excess upon every Degree does not amount at most to the quantity of 3 Feet, so that we may consider all these particular Lines of which the total Distance N ɑ is composed, as insensibly different from the Curviture of a Meridian.

For what remains, as we have above given the position of the Towers of *Nostre Dame de Paris*, and of the Observatory, it will be also easie for us to establish the Distances of these same places in respect of the parallels of *Malvoisine*, and of *Amiens*.

For first, if from GD, which is of 25643 Toyses, there be taken D S, found before of 12795 Toyses, there will remain 12848 Toyses for G S, which is the Distance between *Mareüil*, and the Towers of *Nostre Dame*: This Line G S makes with G E, an Angle of 12° 34′ 30″, toward the *West*, and by consequence also it declines towards the *West* by 13° 00′ 30″. Then having drawn S ɑ, which let be perpendicular to the Meridian of *Mareüil*, and which represents an Arch of the parallel of the Towers of *Nostre Dame*, we have

In the Triangle G ɑ S rectangled at ɑ.

G S 12848 Toyses.
ɑ G S 13° 00′ 30″.
Thence G ɑ 12518 Toyses.
And S ɑ 2892 Toyses.

Second Plate. Then if from G ɑ, which is of 31894 Toyses, be taken G ɑ 12518 Toyses, there remains ɑɑ of 19376 Toyses, for the Distance between the Parallels of *Nostre Dame*, and of *Malvoisine*, which may also be yet further verified by the following Calculation.

In

In the Triangle S D E.

S D E 128° 5′ 30″.
S D 12795 Toyses
D E 8872 Toyses.
Thence E S 19556 Toyses.
And D E S 30° 59′ 30″.
But D E G 39 12 30.
Thence S E G 8 13 00.

But E G declines by 26′ from the *North* towards the *East*, thence E S declines by 7° 47′ from the *North* towards the *West*; and because that the length of this same Line E S is 19556 Toyses, it follows, That the distance between the Parallels of *Nostre Dame*, and of *Malvoisine*, is 19376, as by the former Calculation.

In fine, in the Triangle Z D E.

Z D E is 129° 18′.
Z D is 15757 Toyses.
D E 8871 Toyses.
Thence E Z 18685 Toyses.
And D E Z 29° 08′ 30″.
But D E S is 30 59 20.
Thence S E Z is 01 50 50.

The last Angle S E Z being added to the Declination of the Line E S which was above found of 7° 47′ makes the Declination of E Z of 9° 38′; but the length of this same Line E Z is of 18685 Toyses; thence by Reduction the Distance between the Parallels of *Malvoisine*, and of the Observatory, shall be of 18421 Toyses: And in fine, that between the parallels of *Nostre Dame* and that of the Observatory, shall be of 955 Toyses, 3 Foot.

And tho in all our Observations which we made for determining the Position of divers Lines with respect to the Meridian, we did not at all make use of the Compass (or Magnetical Needle) yet this hindred not, but that we observed the Declination of the Needle in several places principally at *Malvoisine* and at *Bourdon*: The Needle of the Compass which we carried, was 5 inches long, and its Declination at these two places, toward the end of the Summer of the Year 1670, was found to be 1° 30′, from the North toward the West, or thereabout, as we had some little time before observed it at *Paris*, with the same Compass, although at *Paris* the same Needle

Needle in the Year 1666 had no declination sensible, and in the Year 1664 it declined 40' towards the *East*, the variation thereof having been every Year above 20'.

ARTICLE IX.

FOR concluding in fine the Magnitude of a Degree, and by consequence that of the Earth, it remains yet to know what parts of the Meridional Distances we have measured with the Toise of *Paris*, do answer to Minutes and Seconds, considering them as parts of a great Circle which should be described round about the Earth.

'Tis upon this occasion that we are obliged to search in the Heavens the Measure of the Earth, for we must necessarily have recourse to the difference of the Latitudes of the two places established under one and the same Meridian, and by this means come to the knowledg of the Arch of the Heavens comprised between the Zeniths of the said Places, the which Arch is alike to that which we have measured upon the Earth.

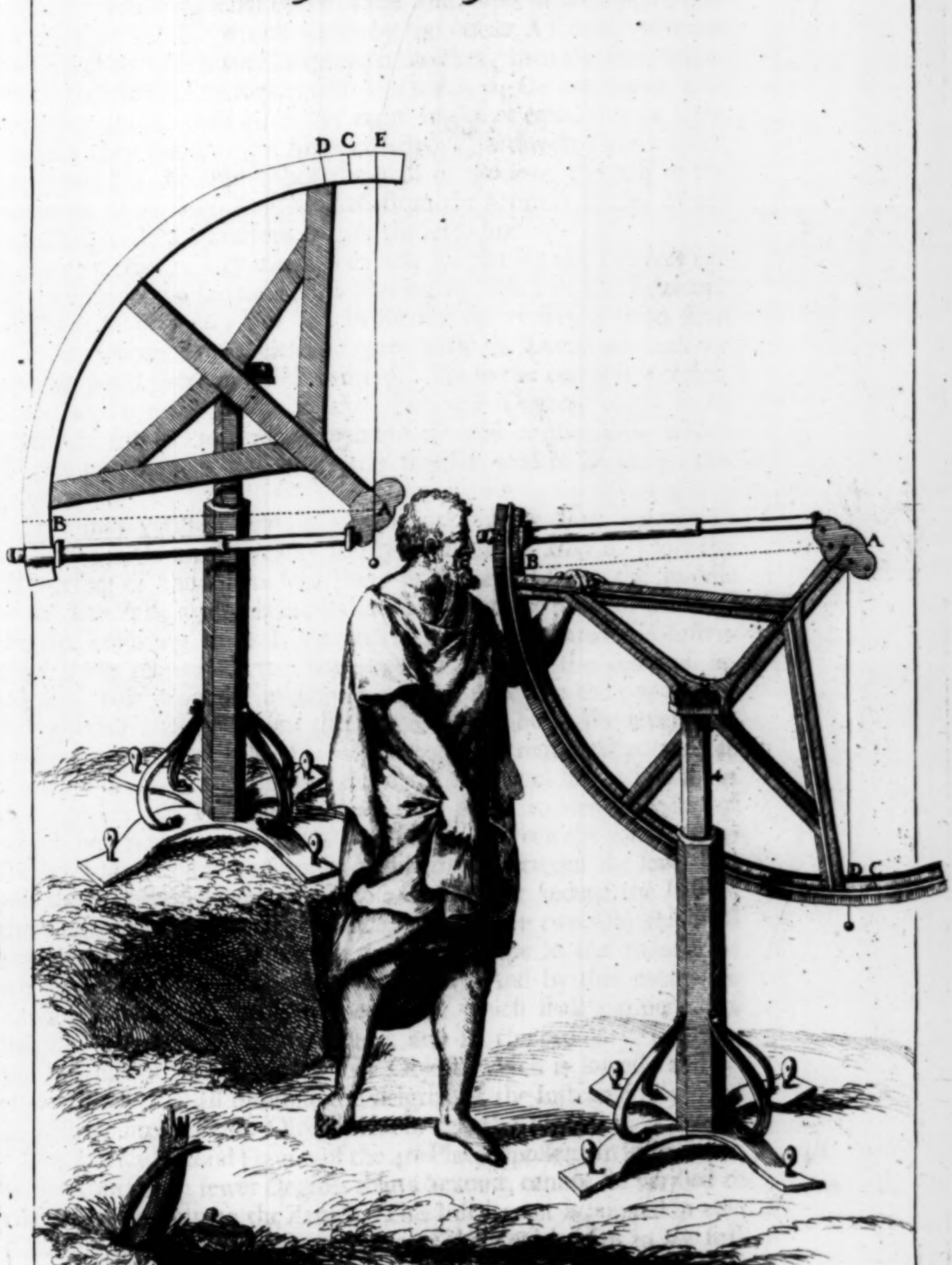
But before we pass to the Celestial Observations, it will be to the purpose to shew after what manner the Instruments were verified with which the observations were made; which is here so much the more necessary, for that the Tellescopes which we made use of might have had some latent defect, which could not be known, but by a particular Proof.

Plate the third. The first Figure of the 3d plate represents the Quadrant fitted upon its Foot in the ordinary manner as for taking of heights, or for directing at an Object far distant towards Edges of the Horizon; but in the 2d Figure the same quadrant is reinverted, turned from the right to the left, and directed at the same Object as before, in such sort, that the plumb line which in the former position was suspended at the Center A, and beat upon the Limb in D, is now hung upon the Limb in E, and beats precisely upon the Center A, the Instrument is also placed upon a place more elevated, to the end that after the Reversing, the Telescope might lie very near in the same line as before, tho in effect it is sufficient that it remain in a Line parallel to the former, as it will always happen if the distance of the Object be so great, that the alteration caused by the reinversion be not at all considerable, or at least if two Objects are directed at, one of which is as much below the other as the Tellescope is altered by the reinversion.

Supposing then that before the reinversion, one has marked upon the Limb of the Quadrant, the point D, where the plumb line beats, and after the reinversion one has also mark'd the point E, where the plumb line is to be hanged, the Point C taken in the middle of the Interval DE shall determine the beginning of the division of the Quadrant, and if after the instrument be put into its former position the plumb line comes to beat upon the point C, the Tellescope sight must

ne;

Plate the Third of the measure of the Earth pa. 24.



necessarily be directed in the level line; so that if by chance they are at first sight so pointed, there will be found no other than the same point before and after the reinversion.

The reason of this method is easie to be comprehended, because without considering what passes in the Telescope, if we suppose that the right line A B (which passes by the center A) tends towards the Object to which the Telescope is directed, then the two Angles which the plumb line makes with the line A B, the one under, and the other above, shall be either right Angles or equal to two right Angles; they shall be right Angles when one has directed in the level, but if one has directed either too high or too low, the half of the difference of the two Angles taken from the greatest Angle, or added to the least, shall restore or give the level line.

This practice is very useful, not only for placing the Degrees upon the limb of an Instrument, following the effect of the Telescope whatever it may be; but 'tis yet further for verifying from time to time, whether the Telescope agree with the Division which we have supposed good and well centred. But to the end this Verification may be made with the more ease, the Degrees ought to be continued from C towards E, even to the end of the limb, which for this purpose ought to be greater than it need to be for 90 Degrees only.

One may verifie a Sextant very near after the same manner as a Quadrant, as we may easily see by considering, that if before the reinversing of the Instrument there be suspended from the middle of the line A B, a plumb line which falls upon the point of the *60th* Degree, counting from B towards D, and afterwards the Instrument being reinversed, the same line hanging on the point of 60 Degrees, falls upon the middle of the line A B. In the one and in the other of these positions the line A B shall be in the level, and by consequence the Telescope ought to have remained pointed at the same distant Object which did mark the level line. But if on the contrary the Telescope be found to point to two Objects, of which one is above the other, the middle between the two shall be the level line. Now the Angle of difference between the level line and the one and the other of those Objects, or indeed the half of the Angle of the appearing distance between the two Objects, shall after be easily measured with a great Telescope in the manner as we measure the Diameters of the Planets: And by this means we know the error of the Instrument, the which shall augment the heigths, if before the reinversment, and in the ordinary position, the Instrument be pointed at that Object which is lowest, and on the contrary it shall diminish the heigths, if the Instrument is found at first pointed at that Object which was the highest.

The first and second Figures of the *4th* Plate represent an Instrument, which containing fewer Degrees than a Sextant, cannot be verified to the level, but only to the Zenith. This Instrument is pointed in two differing manners to the same Star near the Zenith. For in the first Plate 4th.

Figure the plumb falls in D upon the Degrees of the Limb. And in the second as the Instrument is counterturned the same Plumb falls without, and is approached to the Telescope in E. Now it is easie to see that if one draws the line A B from the center A through the middle between the points D and E, marked by the two positions of the plumb Line, it shall determine the place of the Limb where the first Degree of account from the Zenith ought to begin, because that when the Telescope shall be pointed to the Zenith, the line of the plumb shall agree necessarily with the line A B.

This second manner of verifying is general for all sorts of Instruments, but it is difficult and cannot at all times be practised, because it requires a Star which shall be so near the Zenith, that after the Instrument is counterturned, and that it is pointed to this Star, the Plumb may always fall between the point B and the Telescope.

All those Instruments which serve to take heights, and which have an Alidade which one can take away when one will, are easie to be verified. The Instrument ought to be placed in the plain of the Meridian, making it perfectly immovable as if it were fixed against a Wall in such a sort, notwitstanding that the Plumb beating towards the middle of the Limb, leaves on the one and the other side so many Degrees as are necessary for the Observations which are to be made with it. Two fixed Stars are to be made choice of, whereof the one ought to pass on this side, and the other on that side of the Zenith, and of which the difference or the summ of their Declinations do not surpass the number of the Degrees marked upon the Instrument. This being supposed, the two Stars are to be observed with the Telescope upon the Alidade according to the measure which they pass the Meridian, the one towards the North, and the other towards the South; and then provided the Instrument remains immovable, the difference between the two Observations will give exactly the Arch of the Meridian between the parallels of the two Stars, independent from all that could happen on the account of the Telescope of the Alidade. This preparation being made, the Alidade is to be taken off for putting a plumb Line in its place, and one must observe with the Telescope which is fastened to the Instrument, the apparent distance which is between the Zenith and each of these Stars taken in the Meridian, if the Instrument depresses, the summ of the two distances found by this last manner shall be too great; and on the contrary, if it Raises, then it shall be too little in comparison of the total distance found by means the Alidade in such manner, that the half of the difference shall be the Error of the Instrument.

One may make a second Verification by observing one Star only, the distance of which from the Zenith doth not exceed the number of the Degrees of the Instrument to be verified, but in lieu that in the preceeding manner there was no necessity to have compared the Telescope

Telescope of the Instrument with that of the Alidade. It is necessary here that they must be both well adjusted together at one and the same far distant Object. This being supposed, one observes first with the Plumb, and with the Telescope fastned to the Instrument, the Meridional distance between the Zenith and the Star proposed, next one fixes this Instrument in the plain of the Meridian, as in the preceding manner, but in such sort, that it may be counterturned, and that if the Star be towards the South, it returned as 'twere for observing towards the North, and one observes exactly the Degree and Minute of the Limb where the Plumb beats. After this the the Plumb being taken off, one applies the Alidade, with which one observes the Meridional Distance between the Zenith and the Star, counting for this effect the Degree and Minutes which are found between the fiducial line of the Alidade, and the part of the limb where the plumb did beat before. The first distance that was found being compared with this last, shall be too little if the Instrument elevates; and on the contrary, it shall be too big if it depresses in such sort that the half of the difference shall be the error of the Instrument.

After one has known the error of the Instrument, and that one is assured that it comes not but by the Telescope, the shortest and easiest way is to let it alone, and to have regard to it in the Observations; but if one would correct it, this may be done either by displacing the Filaments of the Telescope, or by turning the Object Glass upon its Center; so far as one knows by experience it is necessary for adjusting the Telescope to the Degrees of the Instrument. An Alidade furnisht with its Telescope may be of great help to make this correction; for this purpose one points to one and the same distant Object, as well the Telescope of the Alidade as that of the Instrument. Next, if the error is, for example, of one Minute in elevating, one sets back the Alidade a Minute; or on the contrary, one puts it nearer it, as much if the error be in depressing; and having fastned it in this position, by removing the Instrument all together, one makes the Telescope of this Alidade to stand pointed at the same Object as before; after which you must turn the Object Glass of the Telescope, which is fastned to the Instrument upon its Center, till such time as it be found pointed to the same Object; and by this means one may be assured, that a right line which shall be drawn from the Object by the Center of the Instrument, comes to meet the point B, which we suppose to have been established for the beginning of the decision.

But for avoiding as much as is possible the refractions of the Telescope, care must be taken that the Object Glass be well centred, which may be discovered by making it reflect the Rays of the Sun, because if it be well centred, the little focus which it makes by reflection at a certain distance, will be found exactly in the middle of a much greater round of light. Or else, one may observe that the two Images which the Glass reflects of the same Object, come to unite in the middle of its surface.

After this preparation it will be to the purpose to fasten the Object Glass apart in a Copper Box pierced through its two ends, and perfectly turned round; in which, neverthelefs, it must have a little play in such sort that one may a little thrust it from one side to t'other by three Screws with their heads cut off to hold it steady; and this Box being exactly enchased into the Objective Pinnule, one may make it turn upon its Center, mean while the whole body of the Telescope remains immoveable; and one may observe, that if in making the Object Glass so to turn, the Telescope always remains pointed to the same Object, otherwise the Object Glass must be moved either to the one side or the other.

We thought it necessary to give all these differing ways of verification, to the end that there might remain no doubt as to the great exactness which one ought to look after in Telescopes used for Pinnules or sights of Instruments.

ARTICLE X.

IF the measure of the Earth requires precise and exact Observation, it is principally for that which concerns the difference of Latitudes; because the error of one Minute only amounts to 951 Toyses, which is multiplyed upon the whole as many times as the distance measured is contained in the whole Circumference of the Earth.

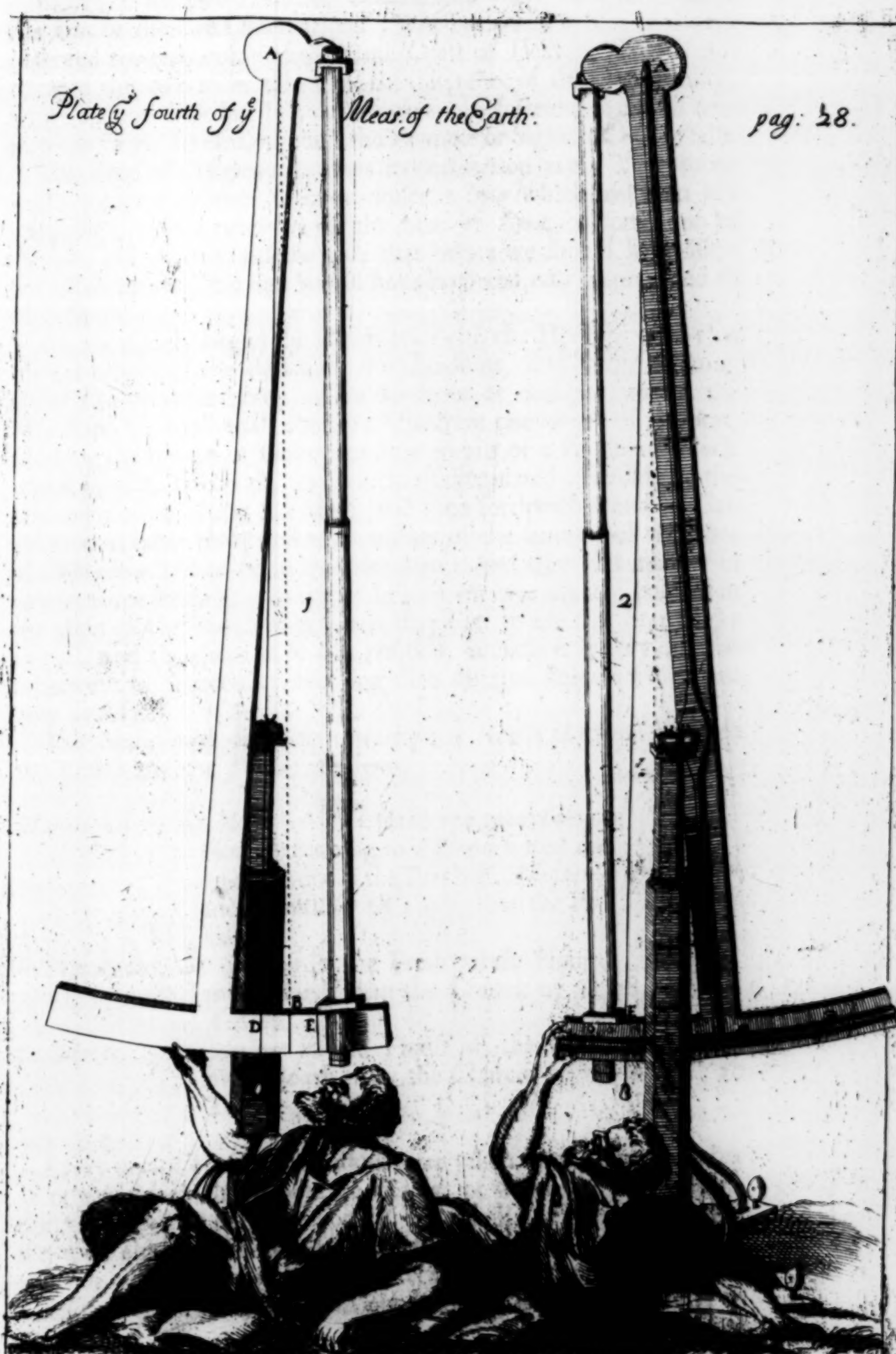
Plate 4*th*. 1*st* and 2*d* Figure. For approaching as much as is possible to the exactness requisite, the great Instrument represented in the fourth Plate was caused to be made; it is of Iron strengthened with pieces upon the Arda of it, as the Quadrant, and covered with Copper at the places necessary. The Limb, which contains not above the 20*th* part of a Circle of ten Foot Radius, is divided by Dragonal Lines even to thirds of Minutes very distinctly.

A Telescope of ten Foot serves for Pinnules or Sights to this Instrument. And because that in the obscurity of the Night one could not see the Filaments that were in the Telescope, they were enlightened by the upper end of the Telescope, or by a hole made on the side.

The Plumb or Perpendicular was secured in a Pipe of Tin, which kept it intirely covered from the Wind, beside that they always observed in a close place, of which the cover or roof was purposely pierced.

For determining with this Instrument the differences of the Latitude of *Malvoisine*, of *Sourdon*, and of *Amiens*, the Star called the *Knee of Cassiopea* was made choice of, which comes to the Meridian at 9 or 10 Degrees of distance from the Zenith towards the North, about 28′ 46″ of time after the Polar Star. A Star more near to the Zenith would have been more difficult to be well observed. And if otherwise it should have been placed between two Zeniths, the error of the Instrument which might not possibly be so

Plate ye fourth of ye Mear: of the Earth. pag: 28.



ſo perfectly diſcovered, would have been doubled in the apparent diſtances of the two Zeniths, becauſe you muſt then have taken the ſum of the two Obſervations. Whereas when a Star is always obſerved towards one and the ſame Coaſt of Heaven, there is nothing in this caſe to be taken but the difference of the Obſervations, which cannot chuſe but be exact, becauſe the Inſtrument is well centered and well divided, though the Pinnules or Sights had been falſe.

The Knee of *Caſſiopea* augments its declination every Year about 20″; we were deſirous to have choſen a Star which had been leſs changing, as had been the bright Star of *Lyra*, or ſome one of *Cygnus*; but we had cauſe to fear, that before we ſhould have made our Obſervations, the Sun would have been too near approached to theſe Stars.

We commonly begun the Obſervations of the Heavens with that of the heigth of the Pole with the Quadrant, and every Evening about two or three hours before the Knee of *Caſſiopea* was in the Meridian, we took with the ſame Quadrant one heigth of this Star, marking the Inſtant of Obſervation by means of a Pendulum Clock which gave half ſeconds, and which was regulated according to the Diurnal motion of the fixt Stars, and then forthwith found by Calculation at what Hour and what Inſtant of the ſame Clock the Knee of *Caſſiopea* ought to be in the Meridian : And after this manner in two or three Evenings, the great Inſtrument was exactly pointed in the plain of the Meridian towards that part where this Star ought to paſs, and then kept it in this poſition, becauſe it is very difficult otherways to ſucceed in obſerving thoſe ſorts of heigths which paſs very ſwiftly.

The Meridional diſtances towards the North obſerved between the *Zenith* and the *Knee of Caſſiopea.*

In *Sept.* 1670.	At *Malvoiſine* in a place at a great Farm-Houſe belonging to *Villeroy* ſeated on an eminence in the Pariſh of *Chauqueil*, more South by 18 Toyſes than the Pavilion.	9°	59′	5″.
In *Sept.* & *Oct.*	At *Sourdon* in the Presbyterate Houſe, more North than the Church by 65 Toyſes.	8	47	8
In *October.*	At *Amiens* in the Houſe of the King, more South than the Church by 75 Toyſes.	8	36	10

Every one of theſe Obſervations were taken from a great number of others, of which we took the middle, of which the whole variation or difference exceeded not 5″. Nor will any one wonder that we were able to come to ſo much exactneſs, if he conſider that it was not without exceeding great precaution, that moreover with a Teleſcope of 10 Foot, one need not want 2″ of pointing exactly to a fixed

a fixed Star. And that in fine on the Instrument that serv'd for this purpose, the third part of a Minute was at least as big and distinct as a whole Minute of the Quadrant above represented. In such sort, that if upon the Quadrant one could determine a quarter of a Minute pretty exactly, and at the same time guess pretty near at $10''$, one might do the same thing here to about $3''$.

Differences of Latitude.

From *Malvoisine* to *Sourdon*	1°	11′	57″.
From *Malvoisine* to *Amiens*	1	22	55.

The time which passed between these Observations required that we should have taken away $1''$ from the first of the Differences, and that in proportion the last should have been diminished by $1\frac{1}{4}''$, but for avoiding a too much affected preciseness, we neglected this Correction.

ARTICLE XI.

Plate the Second.

ALL these Observations being supposed, it will be easie thence to conclude the magnitude of a Degree upon the Earth. For this effect it must be considered, that at *Malvoisine* the Observations of Heaven were made at 18 Toises more towards the South than the Point E. that on the contrary at *Sourdon*, it was at 65 Toyses more towards the North than the Point N. And that by consequence 83 Toyses should be added to the distance of 68347 Toyses, 3 Foot, which are found between the Parallels of *Malvoisine* and of *Sourdon*; in such manner that the difference of 1° 11′ 57″, observ'd by the Heavens, answers upon the Earth to a Meridional distance of 68430 Toyses, 3 Foot, one may thence in fine conclude, That in proportion a Degree shall be of 57064 Toyses, 3 Foot.

The Calculation made by the distance of *Amiens* differs not at all from the former, for the distance between the Parallel of *Nostre Dame d' Amiens*, and that of the Pavilion of *Malvoisine* is of 78907 Toyse; there ought to be taken from the side of *Amiens*, for the place of Observation, 75 Toyses; and on the other side to add the 18 Toyses of *Malvoisine*; then all the compensation made, there will be 78850 Toyses, for the difference of 1° 22′ 55″; and in proportion the degree shall be of 57057 Toyses, which number approaches in such sort to the first, that we were surprised so much the more, that if we had kept account of the Corrections which we have neglected of the differences of Latitude, these two Calculations would have been yet more approaching to each other. It is possible that this is but an effect of chance, since notwithstanding all the exactness we were capable of, we could not answer to two Seconds, and consequently to the value of about thirty two Toyses, upon every observation: We may nevertheless say with some certainty, that we are

not

not very far from the true measure of a degree; though one may come to a yet greater preciseness, by measuring with the same care and with like Instruments a distance much greater than that of *Malvoisine* and *Amiens*. We will fix notwithstanding upon the round Sum of 57060 Toyses for a degree of a great Circle of the Earth.

'Tis here principally, that the measure taken from Pendulums, ought to be imployed, which we have supposed * universal, or at least invariable for every place; and which is to the *Parisian* Toyse, as 881 to 864, because following this proportion, the degree shall be of 55959 universal Toyses, of which every one contains two lengths of a pendulum of Seconds of mean time, so that there wants but 41 of these Toyses upon a whole degree to make up the Round Number of 56000. And by consequence the degree to be of 28 Universal Miles, such as we have determined them. * Artic. 4.

To the end that strangers may participate of this work, without being obliged to have recourse to the length of a Pendulum of Seconds, we shall give the length of a degree, expressed according to the particular Measures of which we could gain the knowledg.

Supposing then	*The Paris Foot, of*	1440 *parts.*
	The Rhein or Leyden Foot	1390.
	The London Foot	1350.
	The Boulogne Foot	1686.
	The Brase of Florence	2580.

A Degree of a Great Circle of the Earth, according to the Measures of divers places will contain

Toyses of the Castle of *Paris*	57060.
Pases of *Boulogne*	58481.
Verges of *Rhein of* 12 *foot each*	29556.
Parisian Leagnes *of* 2000 *Toyses*	$28\frac{1}{2}$.
Midling Leagues *of France of about* 2282 *Toyses*	25.
Marine Leagues of 2853 *Toyses*	20.
English Miles *of* 5000 *Foot each*	$73\frac{7}{111}$.
Florence Miles *of* 3000 *Brasses*	$63\frac{7}{10}$.

The Circumference of the Earth.

Of *Parisian* Toyses 20541600.
Of Leagues of 25 in a degree 9000.
Of *Marine* Leagues 7200.

The Diameter of the Earth.

Of *Parisian* Toyses 6538594.
Of Leagues of 25 in a degree $2864\frac{4}{11}$.
Of *Marine* Leagues $2291\frac{3}{11}$.

It

It may be said, that as we have measured the Globe of the Earth by the top of Mountains, or by places more elevated than the rest, it will follow that a degree, such as we have determined, is bigger than that we should find in going still upon the Sea shore, where it should seem that the Measure ought to be considerably less: But that we may see whether this be so, suppose that the line from *Malvoisine* to *Sourdon*, be in all its length, equally removed from the borders of the Sea about 35 Leagues, and that conformable to the Experiments that have been made upon the Seine, the declivity of Rivers, which cross this Line, be about 5 Foot to a League; this shall make at most but 30 Toyses of Declivity, even to the Sea, and putting about 50 Toyses for the height that our Line might have above the Rivers, we shall find that this Line might be elevated about 80 Toyses above the level of the Sea. Whence it would follow that a Degree upon the Sea would be less above 8 Foot, than that we have measured upon the Land, which is not at all to be considered in this matter.

A Table for the value of a Degree of a great Circle of the Earth; divided into

Minutes and Seconds.

Minutes	Toyses.	Seconds	Toyses.
1	951	1	16
2	1902	2	32
3	2853	3	48
4	3804	4	63
5	4755	5	79
6	5706	6	95
7	6657	7	111
8	7608	8	127
9	8559	9	143
10	9510	10	158½
20	19020	20	317
30	28530	30	475½
40	38040	40	634
50	47550	50	792½
60	57060	60	951

It will not be at all difficult hence to find the differences of the heights of the Pole, for all those places of which we have calculated the * Meridional Distances, because 'tis but changing the said Distances into Minutes and Seconds, according to the value of a Degree.

* Artic. 8.

The

The Differences of the Heights of the Pole

between *Malvoisine* and	*The Observatory of Paris*	19′	22″.
	Nostre Dame of Paris	20	22.
	Mareuil	33	32.
	Clermont	52	00.
	Sourdon	71	52.
	Nostre Dame of Amiens	82	58.
Between *Nostre Dame of Paris*, and *Nostre Dame of Amiens*		62	36.

The height of the Pole at *Paris* in the Garden of the Kings Library, by many observations of the Polar Star made in the Winter Solstices has always been found 48° 53′, you must substract 50″, and you have the height of the Pole of *Paris*, about the Towers of *Nostre Dame* of 48° 52′ 10″, or if one had rather design the middle of *Paris* between the Gates of St. *Martin*, and of St. *James*, which is a little way from St *James* of the *Butchery* or Shambles, the height of the Pole of *Paris* will be 48°, 52′, 20″. And we are certain that if the heights of the Pole be fixed, it will have little change from this, tho in the Observatory one may come to a much greater preciseness: we count not the refractions which the Polar star may have, which will be known in time The height of the Pole of *Nostre Dame* of *Paris* being supposed we establish the following heights of the Pole conformable to the differences here above established.

The Latitudes and height of the Pole

of	*Malvoisine*	48°	31′	48″.
	The *Observatory*	48	51	10.
	Nostre Dame of Paris	48	52	10.
	Mareuil	49	5	20.
	Clermont	49	23	48.
	Sourdon	49	43	40.
	Nostre Dame of Amiens	49	54	46.

The difference of the longitudes of these places require a little more of Calculation than that of the Latitudes, because after we had found in a parallel the distance between the Meridians of two places, we reduced this distance to that which is in the Æquator between those same Meridians which were changed into Minutes and Seconds of a great Circle conformable to the Table above. After this manner we found

Sourdon	More *East* than	*Amiens*	$5'$ $54''$.
Clermont		*Sourdon*	1 9
Mareüil		*Clermont*	0 34.
Mareüil		*Malvoisine*	0 20.
Mareüil		*Paris*	4 37.

Whence 'tis easie to conclude that the difference of Longitude between *Sourdon* and *Malvoisine* is only $1'$ $23''$, which confirms the first thought we had that these two places were very near under the same *Meridian*.

It follows also that *Paris* about the Tower of *Nostre-Dame*, is not above $3''$ more Eastward than *Amiens*. And because that in the Parallel of *Paris* $3''$ amount to 1877 Toyses, one must conclude that *Chaliot*, which may pass for one of the Suburbs of *Paris*, is very near in the same Meridian with *Nostre Dame* of *Amiens*.

It would be advantageous to Astronomy if we knew as exactly the difference of Longitude between the Observatory of *Paris* and *Uramburg*, of which one may account more than two Degrees difference, till such time as by Observation made at the same time in these two places, and compared together, we shall be ascertain'd of the truth.

ARTICLE XII.

WHereas the ordinary method of taking the Level is subject to a correction, upon supposal that the semidiameter of the Earth is known, which according to our Calculation is of 3269298 Toyses 3 Foot; We have judged it significant to give here a Table for the correction of the apparent level, and on that occasion we shall speak concerning refractions which intermingle themselves with these kind of Observations, and which hinder them from being serviceable for the Measure of the Earth.

'Tis known that the true Level requires an equal Distance from the Center of the Earth, yet neverthelesss we ordinarily seek the Level in a streight Line, which goes off from the said Center in the manner of a Tangent, hence it is that the true Level is below the apparent.

If instead of taking the Level on one side only, the observer be placed in the middle between the two points which are to be levell'd, from each of which he is equally distant, he will have in this case no correction to make, because the risings will be equal both on the one side and the other side: but without being foreced to this method since the length of the Semidiameter of the Earth is known, the height of the apparent Level above the true is easily found, provided 'tis known at what distance one is from the Object seen; in the same manner as the bigness of the semidiameter of a Circle being known, and that of a Tangent the excess of the secant without the Circle is found.

A Table

A Table of the Heights of the appearing Level above the true.

Distances.	*Heights of the apparent Level.*		
Toyses.	Feet.	Inches.	Lines.
50	0	0	0 1/3
100	0	0	1 1/3
200	0	0	5
300	0	0	11 2/3
400	0	1	9
500	0	2	9
600	0	3	11
700	0	5	4 1/2
800	0	6	11 1/2
900	0	8	9 1/3
1000	0	11	0
1500	2	0	9
2000	3	8	0
2500	5	8	8 1/2
3000	8	3	0
4000	14	8	0

This Table makes it appear that the heights of the apparent level are not at all considerable under 1000 Toyses of Distance, but beyond this they may cause a sensible error, because they increase considerably, and pretty near, as the squares of the Distances.

Those who know not by experience what advantage one may now receive by using Telescope-sights instead of the common sights, will not fail to say that this Table can be of no use, because they have not yet had an Instrument with which they could distinguish the difference that there is between the apparent level and the true. We can notwithstanding assure them, with our Quadrant, which was not more than of three Foot Radius, or with the Instrument of which we are going to give a description, we determined the level to 18 Inches in a distance of 3000 Toyses, for which, according to the Table, eight Foot and three Inches of correction must be made.

The Description of an Instrument proper for observing the Level.

THE Body of this Instrument which is all of Iron, is composed of two principal Rules. The Rule A B is three Foot long, and two Inches broad, it is strengthned underneath by another Rule, to the middle of which is fixed the stem C D, three Foot Fifth Plate first Fig.

Foot and an half long, and perpendicular to the plain of the Rule A B. This stem is fitted with two pieces set edgewise parallel to each other, and which being covered with a very thin Plate, make a square Tube, within which the plumb line or perpendicular G H is inclosed, which is seen through two Glasses which answer to the two extremities thereof. It has also a third opening at the bottom of the Tube, through which, with ones Finger, the motion of the plumb may be stayed.

Article 5. Upon the plain of the Rule A B is fastned the Telescope E F, which is of the same make with that which we have described for the Quadrant; and tho all the pieces have been already represented in the first Plate, yet we judged it not impertinent to represent it once more in another order, and a bigger size: But that we might not be obliged to repeat the Discourse, we have put to it the same Letters.

A Painters Æsell serves for a support to this Instrument, and for accommodating it to the inequality of the ground, the Rule A B is arched underneath with two bows which bear upon the two pins of the Æsell; that it may be easie to raise or sink the direction of the Telescope as there shall be need, without altering the Æsell; and when the ground happens to be unequal, one may lengthen this or that Foot of it by the means of a rod of Iron which is joyned to it.

With this Instrument the level may he determined at one glance to a very great distance, even much more than is set down in the precedent Table. But there is generally one great obstacle upon the account of refractions, which makes the Objects appear above the line they ought to be seen in. For example, in the second Figure let A be the center of the Earth, B C its ordinary surface, and D I the tops of the Mountains, we are to consider that the Earth is inveloped with an Atmosphere or vaporous Air composed of different Regions, which are more subtil the further they are removed from the Earth, but in such sort that the change is not made all at once, but by Degrees, the visual Ray which comes from a higher place to a lower, as from D to I, which passes obliquely from a more subtil to a more gross Air, is continually bent in its way in proportion as it changes the *medium*, which gives it the position of a curve line, much like that of D F I, but the Eye that is in I, receives the curve Ray as if it were the Tangent I E, in which it sees the Object D. For the same reason if we suppose another eye in D, it sees the Object I in the strait line D G, tangent to the same bended Ray D F B: And supposing that the two tangents I E and D G which are in place of the visual rays cut each other in H, one may imagine that there happens the same thing, as if the two Objects D and I were respectively seen with one only refraction which should be made in H. and which should be equivalent to all those of the true Ray D F I.

For discovering of these refractions, and also for knowing the total value of them which we suppose reduced to the Angle D H E or I H G. the two Angles A I E and A D G ought to have been observed,

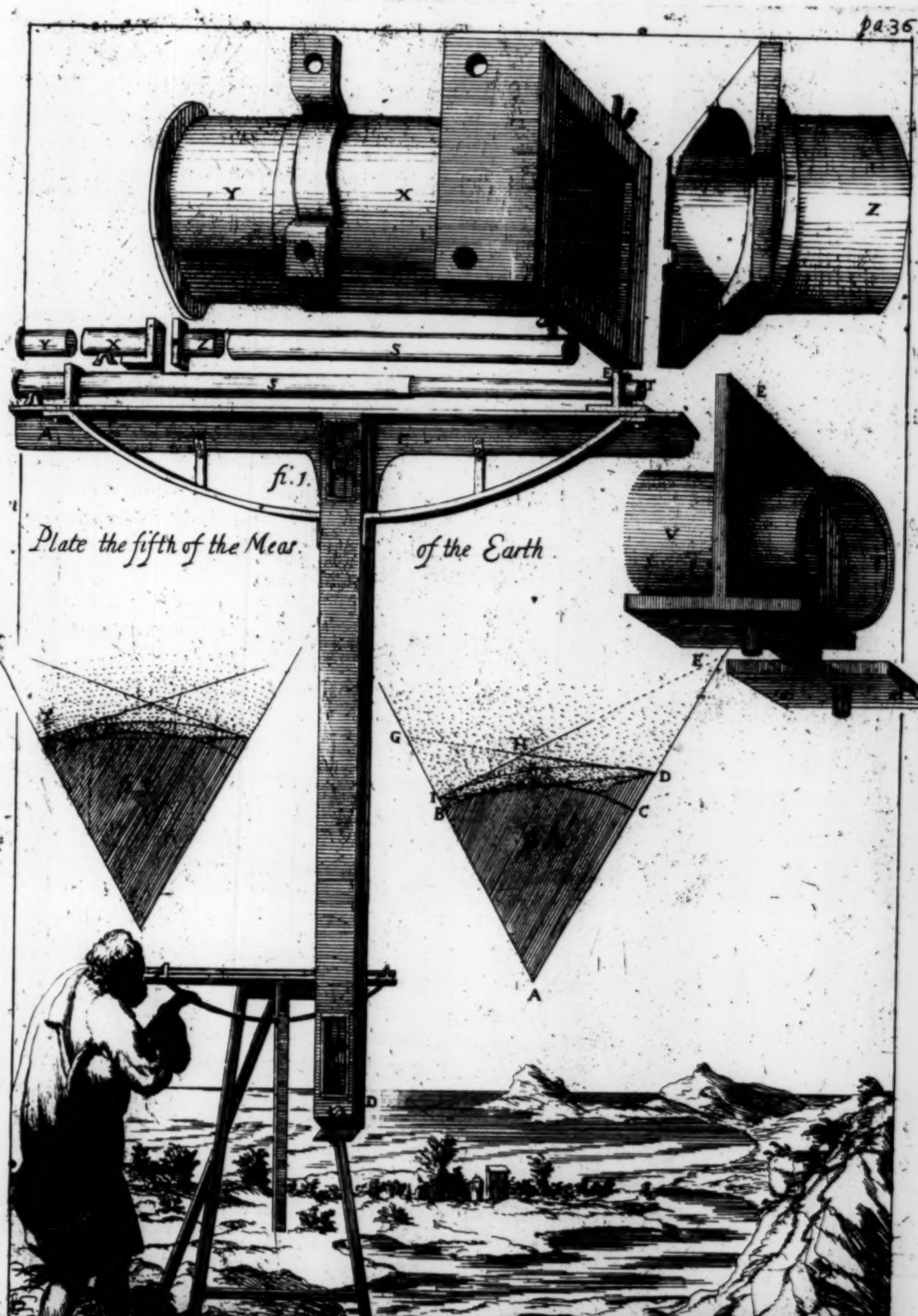


Plate the fifth of the Meas. of the Earth

observed, and moreover the Angle A known, by means of the distance B C or I D. changed into Minutes and Seconds of a great Circle of the Earth; because the excess of these Three Angles above 180 Degrees is the total refraction.

The Third Figure represents Two Mountains of equal height, but so far distant, that the visual Ray cannot pass from the top of one, to the top of the other, without sensibly approaching nearer to the surface of the Earth, and without being consequently broken or refracted in its way, which 'tis not necessary farther to explain. You must always set apart all the irregularities which may happen every moment in the constitution of the Air.

It will be enough for practise, that one can inform ones self of the refraction when there is any, and that otherwise it may be avoided in the Observation of the Level, by contenting ones self with middle stations.

Divers Authors report a thing which we have often tryed; which 'tis convenient to note here, that an Object which at break of the Day has appear'd in the Level, and sometimes a little above it, has afterwards when the Sun is up, appeared below it, and on the contrary after the setting of the Sun, Objects far distant appear'd to be raised so sensibly, that in less than half an Hour their apparent height has been augmented more than Three Minutes.

The cause of these appearances is, that the coolness of the Night condenses the Vapours, which descend to a lower place, leaving the Air of the higher Stations mare pure then in the time of the day, which causes a great Refraction on the contrary when the motion of the Sun has made a part of the Vapours to mount to the more elevated stations, there must be less difference of the *Medium*, and consequently less of Refraction.

We shall add here one Experiment which makes it appear contrary to the Opinion of some Authors, that even at Noon day there remains somewhat of Refraction when the distance is great, and that the visual Ray cannot pass from one place to another without approaching the Earth. The last Summer being on the top of the Towers of *Nostre Dame* of *Paris*, we pointed the quadrant towards the Tower of *Mont Leherie*, and we found that the foot of this Tower was precisely in the apparent Level: This was about Noon in a very Serene time. Some days after at the same Hour, the height of the Tower of *Nostre Dame*, observed from the foot of the Tower of *Montleherie*, appear'd below the Level line 11'. 30''. whereas conformable to the distance of 12796 Toyses, which there are between these two places, this Angle ought to have been 13'. 30''. whence it appears that it had Two Minutes of refraction in the whole.

This experiment shews what exactness one may expect from those who after *Maurolicus* pretend to have found the Magnitude of the Earth, by means of the apparent Level; they suppose that for this purpose, one should chuse a very high Mountain near the Sea shore; and

and having measured the heigth of this Mountain, one tries upon the Sea at what distance the top of it can be seen. But the refractions which are yet greater upon the Sea than upon the Land, render this practice fallacious, because they enable us to discover Objects at a much greater distance than the convexity of the Sea ought to permit, and by consequence make the Earth appear much greater than in effect it is.

ARTICLE XIII.

IT remains now to Examine the differing Opinions touching the Magnitude of the Earth. And because we can say nothing of the Ancients but by Conjecture; we shall begin with *Fernelius* who as we said at the * beginning has estimated a Degree to contain 56746 Toyses.

*Article 1.

It is without doubt surprising, that by a manner so gross as his was, he has approacht so near to that measure which we have concluded on from so many Observations, the place which he took to be the bound of the Degree he had undertaken to measure, was found (by report of the People of the place) as he himself says, at twenty five Leagues of *Paris*, whence he set forth. And besides, this could not be far out of the Road from *Paris* to *Amiens*; because these two Cities are very near, under the same *Meridian*, and that he must have gone directly towards the North; they commonly account 28 Leagues distance between *Paris* and *Amiens*. It was therefore at 3 Leagues on this side of *Amiens*, and by consequence in a place less advanced Northwards by 6'. at least, but the difference of the heights of the Pole of *Paris*, and of *Amiens*, is 62' 36''. whence it follows that *Fernelius* ought not to account above 56' 36''. when he thought he had advanced a whole Degree; so that it must necessarily be that the Error was compensated by the estimate which he made of the Length of the Way.

As for *Snellius*, who gives not above 55021 Toyses, if one considers what we have elsewhere already taken notice of *, that it is founded upon too little a Base; if we add to this, the multitude of his Triangles, the smalness of several Angles, the Correction of three, and sometimes of 4. Minutes, which he was forced to make in the same Triangle; and in fine, 'tis not known by what means he observed the heights of the Pole; we shall less wonder that notwithstanding all his care and pains, he did not succeed so well as *Fernelius*.

*Article 3.

Father *Riccioli* has erred on the other hand, making a Degree to amount to 64363 *Bolnonian* Paces, or to 81 Ancient *Italian* Miles, according as he determins them; but he measured not above a third part of a Degree, which is too little, and besides it is easie to shew what might have deceived him.

Let us imagine, that in the 2d Figure of the 5th Plate, I is the top of the Tower of *Modena*, D the top of the Mountain of *Paterne*, near

near *Boulogne*, and A the Center of the Earth. Father *Riccioli* in his Geography (*lib.5.chap.33.*) assures us that by many observations made at the times which were least suspected for Refractions he always found the Angle A D I of 89° 26′ 13″ 27‴. and the Angle A I D of 90° 15′ 7″ supposing that the two terms I and D were viewed by one strait Ray. the sum of these two Angles makes 179° 41′ 20″ 27‴ and by consequence the Angle A, or the Arch B C, is according to this Observation of 18′ 39″ 33‴; but the distance is of 20016 *Bononian* passes thence by Proportion an intire Degree should be 64363 *Bolougne* passes, which make about 62900. Toises of *Paris*.

This Method which was proposed by *Kepler*, appears so much the more simple, for that there was no need of any Cœlestial Observation, and that it supposes only that the Plumb or Perpendicular tends directly to the Center of the Earth, which we have also supposed. But we may demand of Father *Riccioli*, how he could be assured that in his Observations, he had not any thing of Refraction. It was, says he, at Noon, in places very high elevated. But besides, that one of those Places is much higher then the other; the following Experiment joyned to what we have related before, will make one see what Judgment ought to be made of this Method.

In the Month of *August* of the year 1669. the Top of the Hillock of *Mareüil* observed at Noon, from the foot of the Tower of *Montleherie*, appear'd below the Level 8′ 20″; and some days after at the same hour, the foot of the Tower of *Montlehery* reciprocally observ'd from the Top of the Hillock of *Mareüil*, was found below the Level 13′ 40″. If there had been no Refraction, these two little Angles together would have made the Angle at the Center of the Earth, between *Montlehery* and *Mareüil* of 22′, but the distance is 25643. Toyses: thence in Proportion a Degree should be 69935. Toyses, which will exceed very much, not only the greatness which we have determined by the Heavens; but even that which Father *Riccioli* has found. The Measure without doubt will yet come forth much bigger in respect to two Objects, that shall be further distant then *Mareüil* and *Montlehery*: In such sort that 'tis evident that this method ought to be intirely rejected as fallacious and uncertain.

It may be said, That Father *Riccioli*, understanding well what Refractions would do, did not wholy content himself with this method; but that he did verify it by Cœlestial Observations. But after what manner soever it is in *Italy*, where the Refractions possibly are not so great as here; We have not at all found that the Observations made for the Measure of the Earth, by the means of the Level did agree with those of the Heavens, which we can confirm by divers like Examples to those which we have produced: As one may see in the Geography of the said Author, (*Lib. 5. cap. 27.*) that of the two Observations of the Heavens, one of which gave him 19′ 19″, and the other 21′ 16″, of apparent distance between the Zenith of

Ferrara

Ferrara, and that of the Mountain of *Paterne*, he made choice of the first, as of that which agreed best with his Calculation; whereas, if he had followed the second Observation, we should have found very little difference between us.

Geogr. Reform. l. 5 c. 37. The same Author for the last proof of his Opinion, says, That the distance from *Avignon* to *Lyons*, taken out of the *Itineraries*, accords perfectly with the difference of the heights of the Pole of those two Cities at the rate of 81. ancient Miles for one Degree conformable to his Opinion. It were to be wisht that one knew the just Distance between *Lyons* and *Avignon*; and likewise, that one had to that aded the distance from *Chaalons* on the *Saone*, for one should then have a line of many Degrees almost in a Meridian. Nevertheless one may answer Father *Riccioli*, that the distances reckoned by the Itineraries which he cites, were not measured with exactness enough for the Measure of the Earth, and that he will have a considerable difference between one Itinerary distance, taken in following the great Road, and that which might be measured in the shortest line. Of these Itineraries, that which is attributed to the Emperor *Antoninus*, but which do's often pass under the Name of *Antonius Augustus*, is full of considerable faults; not giving always the same distance between the same two places, as one may see in comparing the Road from *Millan* to *Arles*, with that from *Millan* to *Vienna*. The second Itinerary, which is that of *Bordeaux* and of *Hierusalem*, seems to be the work of some particular Person, who had described his own Travels. And a little Examination will shew that 'tis different from the first in several places, and that the particular distances of several Places between *Arles* and *Millan*, are not at all found to be the same. So that to conclude 'tis not in the least reasonable to regard such kind of Testimonies against a measure exactly taken.

ERRATA.

PAge 1. l. 25. r. *the*. l. 31. r. *to*. p. 2. l. 41. r. *Alcmar*. p. 3. l. 6. r. *for*. p. 4. l. 30. r. *five*. p. 8. l. 11. r. *fifth*. p. 12. l. 19. 21658. p. 13. l. 4. r. 3″. 14‴. l. 34. r. 42°. 27′. 30″. l. 35. r. 49°. 24′. 30″. p. 16. l. 35. r. *this*. p. 18. l. 16. r. *G I*. p. 19. l. 3. r. *Amiens*. l. ult. r. 9073. p. 23. l. 4. r. 8871. l. 16. r. 11757. p. 27. l. 8. r. *be turned*. p. 28. l. 25. r. *Area*. p. 30. l. 8. r. 1°.

FINIS.